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5.5 DATA COMPRESSION

- ▶ *introduction*
- ▶ *run-length coding*
- ▶ *Huffman compression*

Data compression

Compression reduces the size of a file:

- To save **space** when storing it.
- To save **time** when transmitting it.
- Most files have lots of redundancy.

Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, ...

“ Everyday, we create 2.5 quintillion bytes of data—so much that 90% of the data in the world today has been created in the last two years alone. ” — IBM report on big data (2011)

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: GZIP, BZIP, 7z.
- Archivers: PKZIP.
- File systems: NTFS, HFS+, ZFS.



Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.



Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.
- Skype.



Databases. Google, Facebook,



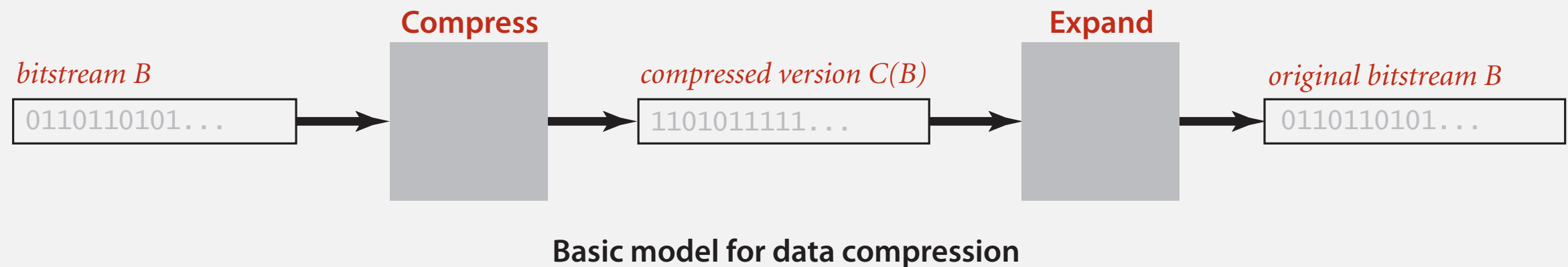
Lossless compression and expansion

Message. Binary data B we want to compress.

Compress. Generates a "compressed" representation $C(B)$.

Expand. Reconstructs original bitstream B .

uses fewer bits (you hope)



Compression ratio. Bits in $C(B)$ / bits in B .

Ex. 50–75% or better compression ratio for natural language.

Food for thought

Data compression has been omnipresent since antiquity:

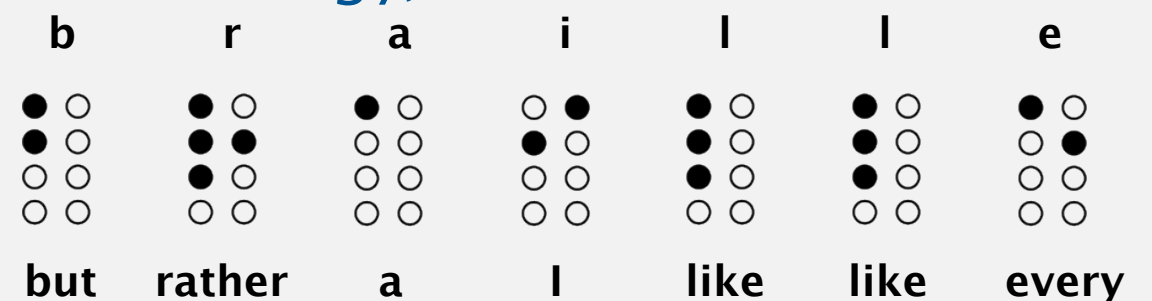
- Number systems.
- Natural languages.
- Mathematical notation.



$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

has played a central role in communications technology,

- Grade 2 Braille.
- Morse code.
- Telephone system.



and is part of modern life.

- MP3.
- MPEG.



Q. What role will it play in the future?

Data representation: genomic code

Genome. String over the alphabet { A, C, T, G }.

Goal. Encode an N -character genome: A T A G A T G C A T A G . . .

Standard ASCII encoding.

- 8 bits per char.
- $8N$ bits.

char	hex	binary
A	41	01000001
C	43	01000011
T	54	01010100
G	47	01000111

Two-bit encoding.

- 2 bits per char.
- $2N$ bits.

char	binary
A	00
C	01
T	10
G	11

Fixed-length code. k -bit code supports alphabet of size 2^k .

Amazing but true. Some genomic databases in 1990s used ASCII.

Reading and writing binary data

Binary standard input and standard output. Libraries to read and write **bits** from standard input and to standard output.

```
public class BinaryStdIn
```

```
boolean readBoolean()      read 1 bit of data and return as a boolean value
```

```
char readChar()            read 8 bits of data and return as a char value
```

```
char readChar(int r)       read r bits of data and return as a char value
```

[similar methods for byte (8 bits); short (16 bits); int (32 bits); long and double (64 bits)]

```
boolean isEmpty()          is the bitstream empty?
```

```
void close()               close the bitstream
```

```
public class BinaryStdOut
```

```
void write(boolean b)      write the specified bit
```

```
void write(char c)         write the specified 8-bit char
```

```
void write(char c, int r)  write the r least significant bits of the specified char
```

[similar methods for byte (8 bits); short (16 bits); int (32 bits); long and double (64 bits)]

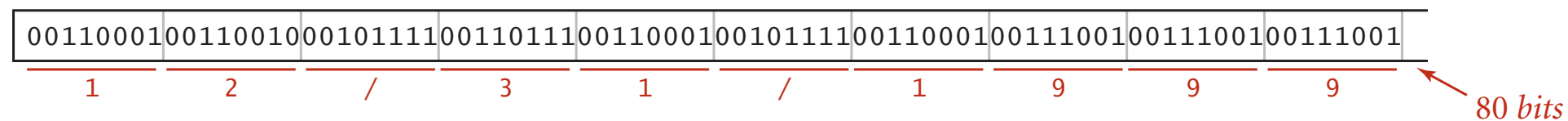
```
void close()               close the bitstream
```


Writing binary data

Date representation. Three different ways to represent 12/31/1999.

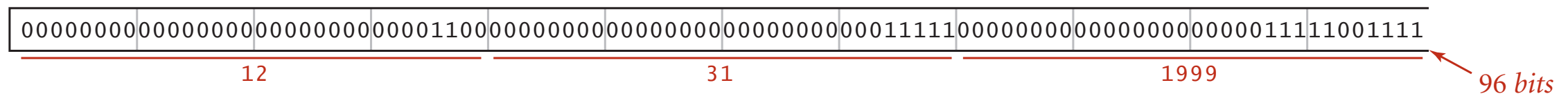
A character stream (StdOut)

```
StdOut.print(month + "/" + day + "/" + year);
```



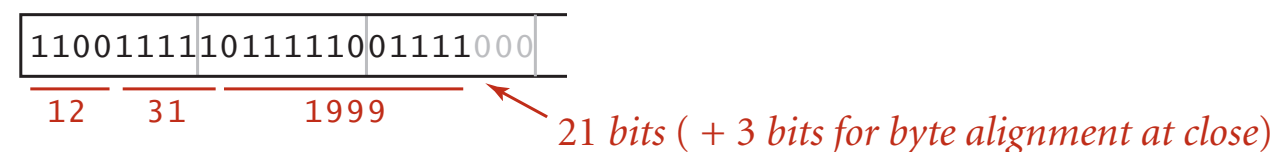
Three ints (BinaryStdOut)

```
BinaryStdOut.write(month);  
BinaryStdOut.write(day);  
BinaryStdOut.write(year);
```



A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)

```
BinaryStdOut.write(month, 4);  
BinaryStdOut.write(day, 5);  
BinaryStdOut.write(year, 12);
```



Binary dumps

Q. How to examine the contents of a bitstream?

Standard character stream

```
% more abra.txt  
ABRACADABRA!
```

Bitstream represented as 0 and 1 characters

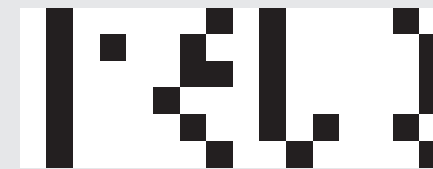
```
% java BinaryDump 16 < abra.txt  
0100000101000010  
0101001001000001  
0100001101000001  
0100010001000001  
0100001001010010  
0100000100100001  
96 bits
```

Bitstream represented with hex digits

```
% java HexDump 4 < abra.txt  
41 42 52 41  
43 41 44 41  
42 52 41 21  
12 bytes
```

Bitstream represented as pixels in a Picture

```
% java PictureDump 16 6 < abra.txt
```



← 16-by-6 pixel
window, magnified

96 bits

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	“	#	\$	%	&	‘	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

Hexadecimal to ASCII conversion table

Universal data compression

[US Patent 5,533,051](#) on "Methods for Data Compression", which is capable of compression **all** files.

[Slashdot](#) reports of the Zero Space Tuner™ and BinaryAccelerator™.

*“ ZeoSync has announced a breakthrough in data compression that allows for 100:1 lossless compression of **random** data. If this is true, our bandwidth problems just got a lot smaller.... ”*

Universal data compression

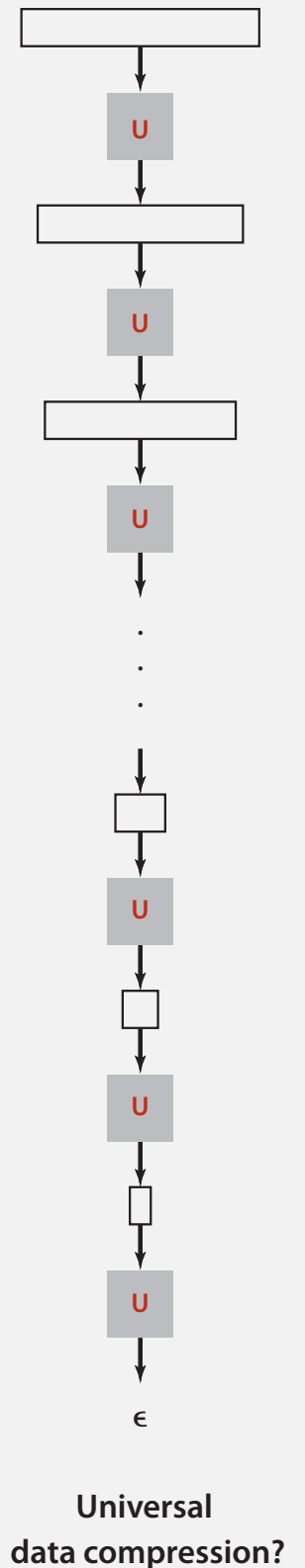
Proposition. No algorithm can compress every bitstring.

Pf 1. [by contradiction]

- Suppose you have a universal data compression algorithm U that can compress every bitstream.
- Given bitstring B_0 , compress it to get smaller bitstring B_1 .
- Compress B_1 to get a smaller bitstring B_2 .
- Continue until reaching bitstring of size 0.
- Implication: all bitstrings can be compressed to 0 bits!

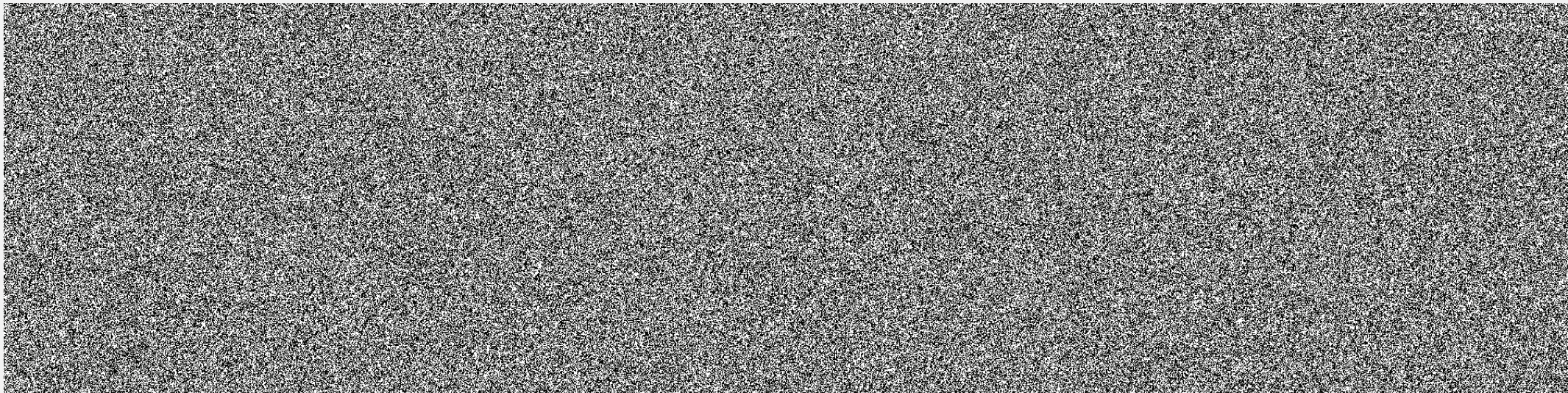
Pf 2. [by counting]

- Suppose your algorithm that can compress all 1,000-bit strings.
- 2^{1000} possible bitstrings with 1,000 bits.
- Only $1 + 2 + 4 + \dots + 2^{998} + 2^{999}$ can be encoded with ≤ 999 bits.
- Similarly, only 1 in 2^{499} bitstrings can be encoded with ≤ 500 bits!



Undecidability

```
% java RandomBits | java PictureDump 2000 500
```



1000000 bits

A difficult file to compress: one million (pseudo-) random bits

```
public class RandomBits
{
    public static void main(String[] args)
    {
        int x = 11111;
        for (int i = 0; i < 1000000; i++)
        {
            x = x * 314159 + 218281;
            BinaryStdOut.write(x > 0);
        }
        BinaryStdOut.close();
    }
}
```


Rdenudcany in Enlgsih Inagugae

Q. How mcuh rdenudcany is in the Enlgsih Inagugae?

“ ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and sengeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in menieng. ” — Graham Rawlinson

A. Quite a bit.



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- ▶ *run-length coding*
- ▶ *Huffman compression*

Run-length encoding

Simple type of redundancy in a bitstream. Long runs of repeated bits.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1
40 bits

Representation. 4-bit counts to represent alternating runs of 0s and 1s:
15 0s, then 7 1s, then 7 0s, then 11 1s.

1 1 1 1 0 1 1 1 0 1 1 1 1 0 1 1 ← 16 bits (instead of 40)
15 7 7 11

Q. How many bits to store the counts?

A. We'll use 8 (but 4 in the example above).

Q. What to do when run length exceeds max count?

A. If longer than 255, intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

Run-length encoding: Java implementation

```
public class RunLength
{
```

```
    private final static int R    = 256;
    private final static int lgR = 8;
```

← maximum run-length count

← number of bits per count

```
    public static void compress()
    { /* see textbook */ }
```

```
    public static void expand()
    {
```

```
        boolean bit = false;
```

```
        while (!BinaryStdIn.isEmpty())
```

```
        {
```

```
            int run = BinaryStdIn.readInt(lgR);
```

← read 8-bit count from standard input

```
            for (int i = 0; i < run; i++)
```

```
                BinaryStdOut.write(bit);
```

← write 1 bit to standard output

```
            bit = !bit;
```

```
        }
```

```
        BinaryStdOut.close();
```

← pad 0s for byte alignment

```
    }
```

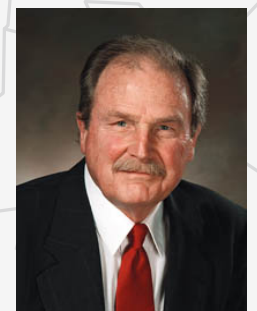
```
}
```




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- ▶ *Huffman compression*



David Huffman

Variable-length codes

Use different number of bits to encode different chars.

Ex. Morse code: • • • — — — • • •

Issue. Ambiguity.

S O S ?

V 7 ?

I A M I E ?

E E W N I ?

In practice. Use a medium gap to separate codewords.

codeword for S is a prefix
of codeword for V

Letters		Numbers	
A	• —	1	• — — — —
B	— • • •	2	• • — — —
C	— • — •	3	• • • — —
D	— • •	4	• • • • —
E	•	5	• • • • •
F	• • — •	6	— • • • •
G	— — •	7	— — • • •
H	• • • •	8	— — — • •
I	• •	9	— — — — •
J	• — — —	0	— — — — —
K	— • —		
L	• — • •		
M	— —		
N	— •		
O	— — —		
P	• — — •		
Q	— — • —		
R	• — •		
S	• • •		
T	—		
U	• • —		
V	• • • —		
W	• — —		
X	— • • —		
Y	— • — —		
Z	— — • •		

Variable-length codes

Q. How do we avoid ambiguity?

A. Ensure that no codeword is a **prefix** of another.

Ex 1. Fixed-length code.

Ex 2. Append special stop char to each codeword.

Ex 3. General prefix-free code.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Compressed bitstring

011111110011001000111111100101 ← 30 bits
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Compressed bitstring

110001111010111100110001111101 ← 29 bits
A B R A C A D A B R A !

Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

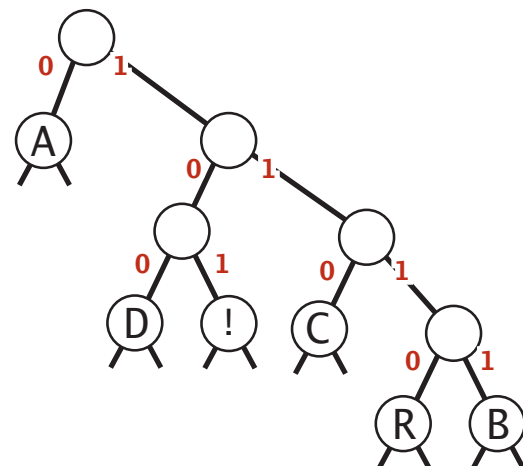
A. A binary trie!

- Chars in leaves.
- Codeword is path from root to leaf.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Trie representation



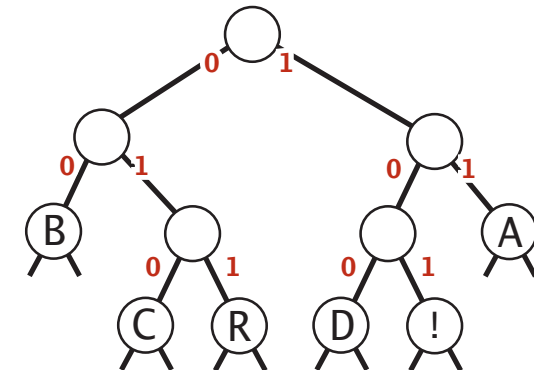
Compressed bitstring

011111110011001000111111100101 ← 30 bits
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Trie representation



Compressed bitstring

11000111101011100110001111101 ← 29 bits
A B R A C A D A B R A !

Prefix-free codes: compression and expansion

Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.

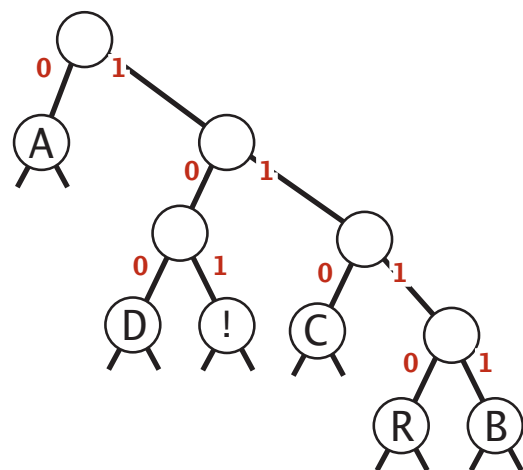
Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, print char and return to root.

Codeword table

key	value
!	101
A	0
B	1111
C	110
D	100
R	1110

Trie representation



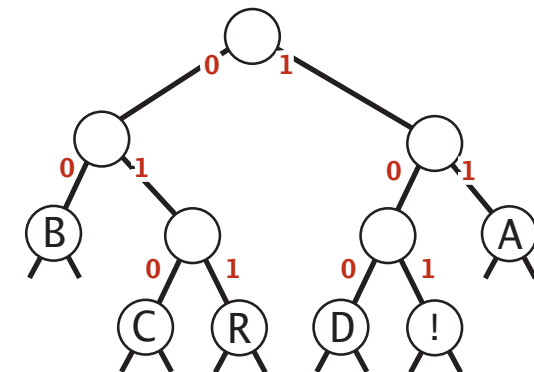
Compressed bitstring

011111110011001000111111100101 ← 30 bits
A B RA CA DA B RA !

Codeword table

key	value
!	101
A	11
B	00
C	010
D	100
R	011

Trie representation



Compressed bitstring

11000111101011100110001111101 ← 29 bits
A B R A C A D A B R A !

Huffman coding overview

Dynamic model. Use a custom prefix-free code for each message.

Compression.

- Read message.
- Built **best** prefix-free code for message. How?
- Write prefix-free code (as a trie) to file.
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code (as a trie) from file.
- Read compressed message and expand using trie.

Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
```

```
    private final char ch;    // used only for leaf nodes
    private final int freq;    // used only for compress
    private final Node left, right;
```

```
    public Node(char ch, int freq, Node left, Node right)
    {
        this.ch    = ch;
        this.freq   = freq;
        this.left   = left;
        this.right  = right;
    }
```

← initializing constructor

```
    public boolean isLeaf()
    { return left == null && right == null; }
```

← is Node a leaf?

```
    public int compareTo(Node that)
    { return this.freq - that.freq; }
```

← compare Nodes by frequency
(stay tuned)

```
}
```


Prefix-free codes: expansion

```
public void expand()
{
```

```
    Node root = readTrie();
```

```
    int N = BinaryStdIn.readInt();
```

```
    for (int i = 0; i < N; i++)
    {
```

```
        Node x = root;
```

```
        while (!x.isLeaf())
```

```
        {
```

```
            if (!BinaryStdIn.readBoolean())
```

```
                x = x.left;
```

```
            else
```

```
                x = x.right;
```

```
        }
```

```
        BinaryStdOut.write(x.ch, 8);
```

```
    }
```

```
    BinaryStdOut.close();
```

```
}
```

← read in encoding trie

← read in number of chars

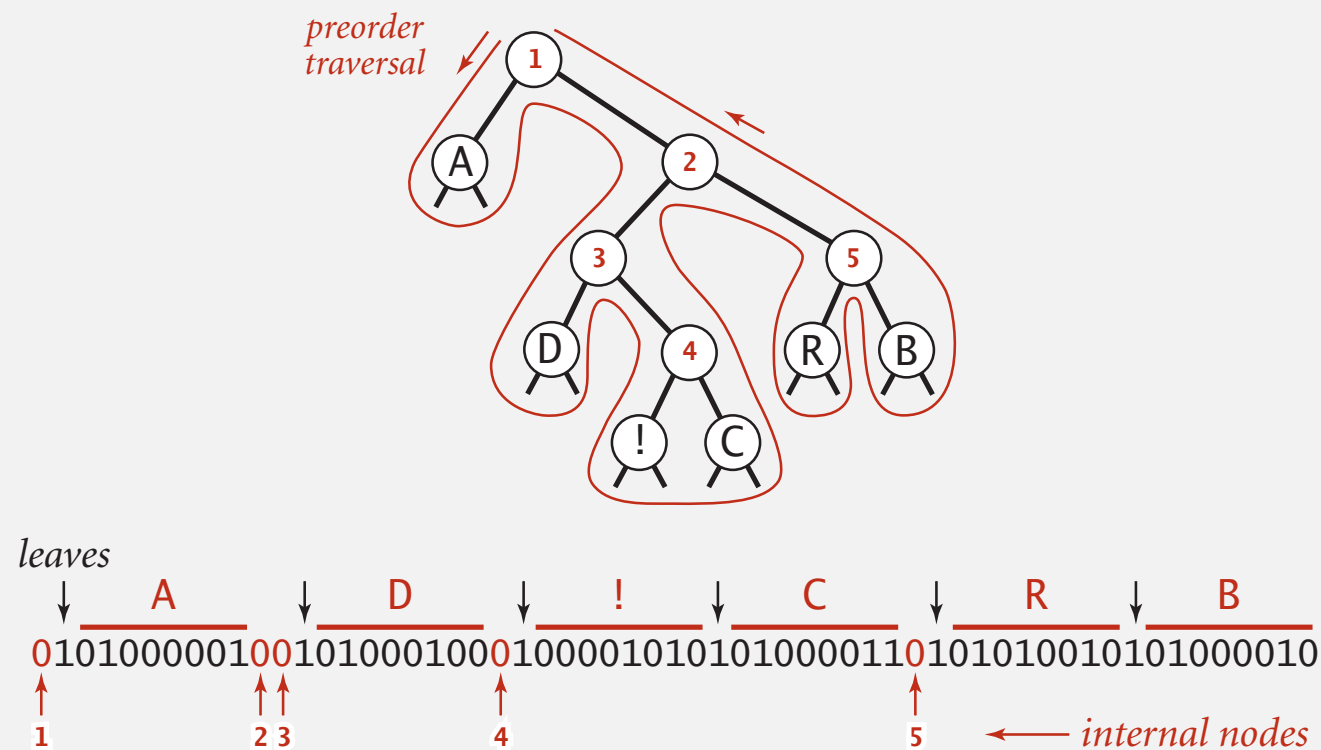
← expand codeword for i^{th} char

Running time. Linear in input size N .

Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal of trie; mark leaf and internal nodes with a bit.



Using preorder traversal to encode a trie as a bitstream

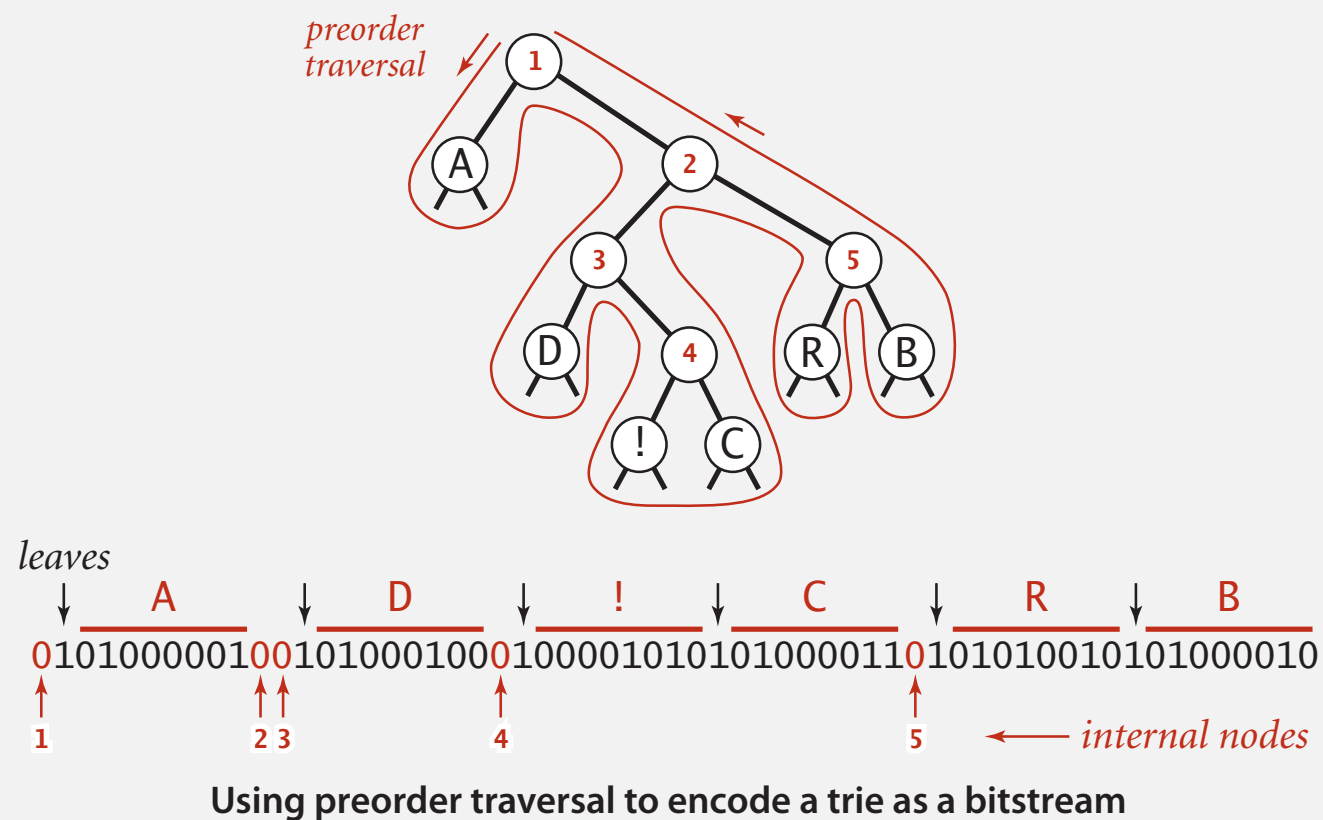
```
private static void writeTrie(Node x)
{
    if (x.isLeaf())
    {
        BinaryStdOut.write(true);
        BinaryStdOut.write(x.ch, 8);
        return;
    }
    BinaryStdOut.write(false);
    writeTrie(x.left);
    writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

Prefix-free codes: how to transmit

Q. How to read in the trie?

A. Reconstruct from preorder traversal of trie.



```
private static Node readTrie()
{
    if (BinaryStdIn.readBoolean())
    {
        char c = BinaryStdIn.readChar(8);
        return new Node(c, 0, null, null);
    }
    Node x = readTrie();
    Node y = readTrie();
    return new Node('\0', 0, x, y);
}
```

arbitrary value
(value not used with internal nodes)

Shannon-Fano codes

Q. How to find best prefix-free code?

Shannon-Fano algorithm:

- Partition symbols S into two subsets S_0 and S_1 of (roughly) equal freq.
- Codewords for symbols in S_0 start with 0; for symbols in S_1 start with 1.
- Recur in S_0 and S_1 .

char	freq	encoding
A	5	0...
C	1	0...

S_0 = codewords starting with 0

char	freq	encoding
B	2	1...
D	1	1...
R	2	1...
!	1	1...

S_1 = codewords starting with 1

Problem 1. How to divide up symbols?

Problem 2. Not optimal!

Huffman algorithm demo

- Count frequency for each character in input.



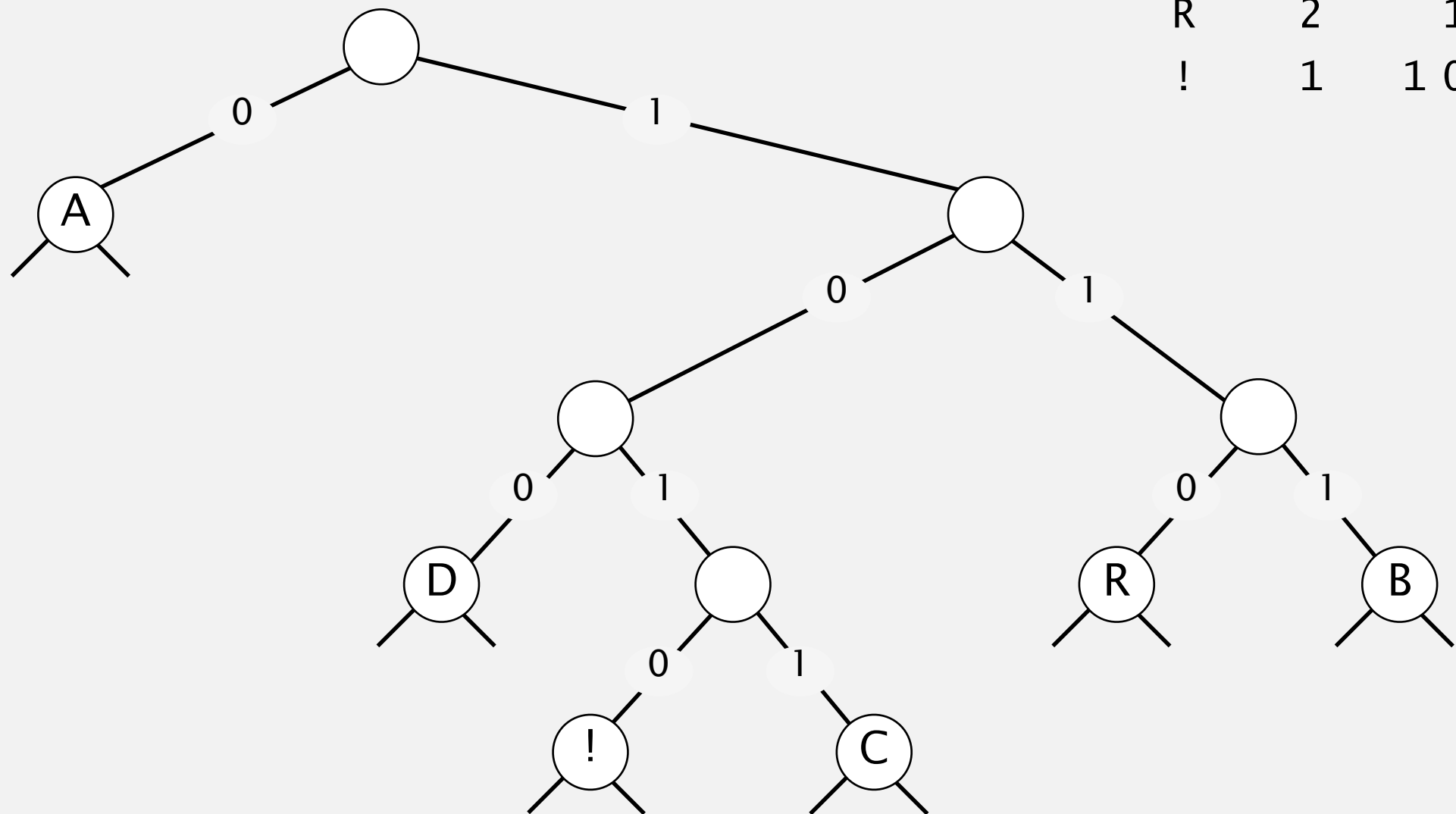
char	freq	encoding
A	5	
B	2	
C	1	
D	1	
R	2	
!	1	

input

A B R A C A D A B R A !

Huffman algorithm demo

char	freq	encoding
A	5	0
B	2	1 1 1
C	1	1 0 1 1
D	1	1 0 0
R	2	1 1 0
!	1	1 0 1 0



Huffman codes

Q. How to find best prefix-free code?

Huffman algorithm:

- Count frequency $\text{freq}[i]$ for each char i in input.
- Start with one node corresponding to each char i (with weight $\text{freq}[i]$).
- Repeat until single trie formed:
 - select two tries with min weight $\text{freq}[i]$ and $\text{freq}[j]$
 - merge into single trie with weight $\text{freq}[i] + \text{freq}[j]$

Applications:



Constructing a Huffman encoding trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
```

```
    MinPQ<Node> pq = new MinPQ<Node>();
    for (char i = 0; i < R; i++)
        if (freq[i] > 0)
            pq.insert(new Node(i, freq[i], null, null));
```

← initialize PQ with
singleton tries

```
    while (pq.size() > 1)
    {
        Node x = pq.delMin();
        Node y = pq.delMin();
        Node parent = new Node('\0', x.freq + y.freq, x, y);
        pq.insert(parent);
    }
```

← merge two
smallest tries

```
    return pq.delMin();
```

```
}
```

↑
not used for
internal nodes

↑
total frequency

↑ ↑
two subtrees

Huffman encoding summary

Proposition. [Huffman 1950s] Huffman algorithm produces an optimal prefix-free code.

Pf. See textbook.

↑
no prefix-free code
uses fewer bits

Implementation.

- Pass 1: tabulate char frequencies and build trie.
- Pass 2: encode file by traversing trie or lookup table.

Running time. Using a binary heap $\Rightarrow N + R \log R$.

↑ ↑
input alphabet
size size

Q. Can we do better? [stay tuned]

Statistical methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling produces better compression.
- Decoding must start from beginning.
- Ex: LZW.